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(54) Title: A METHOD FOR MODELING AN ARBITRARY WELL PATH IN A HYDROCARBON RESERVOIR USING ADAPTIVE MESHING

(57) **Abstract:** Methods and systems for adapting an unstructured mesh to a modeled space. In one embodiment, a surface is defined in the modeled space, intersections of the surface with the elements of the mesh are determined, and new elements within the intersected elements are defined such that one or more of the faces of the new elements are substantially coincident with the surface. For each element which is intersected by the surface, a plurality of points at which the edges of the element are intersected is determined. For each point of intersection, a new node is located at that point and two new elements which incorporate the new node are created. This process is performed for each of the points of intersection so that the intersected element is subdivided into an new elements which include faces that lie substantially on the moduled surface.

A METHOD FOR MODELING AN ARBITRARY WELL PATH
IN A HYDROCARBON RESERVOIR USING ADAPTIVE MESHING

TECHNICAL FIELD OF THE INVENTION

[1]   This invention relates generally to methods and systems
      for modeling physical systems using finite element
      analysis and, more specifically, to methods and systems
      for adapting a finite element mesh to an arbitrary
      surface such as an arbitrary well path trajectory in a
      hydrocarbon reservoir.

BACKGROUND OF THE INVENTION

[2]   Physical systems can be modeled mathematically to
      simulate their behavior under certain conditions. There
      are a wide variety of means to model these systems,
      ranging from the very simplistic to the extremely
      complicated. One of the more complicated means to model
      physical systems is through the use of finite element
      analysis. As the name implies, finite element analysis
      involves the representation of individual, finite
      elements of the physical system in a mathematical model
      and the solution of this model in the presence of a
      predetermined set of boundary conditions.

[3]   In finite element modeling, the region that is to be
      analyzed is broken up into sub-regions called elements.
      This process of dividing the region into sub-regions may
      be referred to as discretization or mesh generation. The
      region is represented by functions that are defined over
      each element. This generates a number of local functions
      that are much simpler than those which would be required

to represent the entire region. The next step in the process is to analyze the response for each element. This is accomplished by building a matrix that defines the properties of the various elements within the region and a vector that defines the forces acting on each element in the structure. Once all the element matrices and vectors have been created, they are combined into a matrix equation. This equation relates nodal responses for the entire structure to nodal forces. After applying boundary conditions, the matrix equation can be solved to obtain unknown nodal responses. Intra-element responses can be interpolated from nodal values using the functions which were defined over each element.

[4]   As indicated above, finite element modeling involves the creation of a mesh of finite elements. The elements are defined by nodes within the problem space. The nodes are simply points in space. The lines between the nodes are referred to as "edges." The mesh is typically a structured mesh. In other words, the mesh is defined in three dimensions so that the elements within the problem space are hexahedrons. For example, they may be cubes or rectangular prisms. (Equivalently, in two dimensional problems, the elements would be rectangles – see FIGURE 1 for an illustration of these elements.) The edges of the hexahedrons are coincident with the edges between the nodes of the mesh. In a simple model, the nodes of the mesh may be regularly spaced to define cubic elements. It is not necessary, however, for the nodes to be evenly spaced in all finite element models. A variety of different sizes and shapes of hexahedral elements can be defined within a single mesh.

[5] One of the problems with meshes which use hexahedral
elements is that they do not lend themselves well to
complex geometries which may need to be modeled. In
other words, it may be difficult to place nodes
sufficiently close to irregular surfaces to accurately
determine the solution to the problem along those
surfaces. Viewed another way, it is difficult to fill an
irregularly shaped object with regularly shaped boxes.
This situation is further complicated by limitations on
the minimum distance between nodes in a structured mesh.
Even when additional nodes elements can be defined in the
mesh to improve the accuracy of the model, the inclusion
of these additional elements causes the resources needed
to solve the resulting model to increase. If increased
accuracy is desired, an increased number of elements, as
well as greater computing power, is required.

[6] One way to reduce the cost associated with increasing the
accuracy of a finite element model is to use an
unstructured mesh. In an unstructured mesh, the elements
which comprise the mesh are not constrained to fit within
a predetermined structural scheme. For example, the
elements are not required to be selected from a finite
set of predefined element shapes (e.g., hexahedra) or
sizes (e.g., fractions of an initial size.) More
particularly, it may be convenient to employ a mesh that
comprises simplex elements which have no predefined
constraints. Because the faces of the simplex elements
need not be parallel to predetermined planes (e.g., the
X-Y, Y-Z or X-Z planes,) they can be oriented to follow
the contours of a surface that is being modeled and may
therefore achieve accuracy that is on par with a

structured mesh. As a result, an unstructured mesh that conforms to the geometry is simpler to create.

[7] Although unstructured meshes may be capable of more accurately modeling irregular surfaces than structured meshes, it may nevertheless be difficult to construct an unstructured mesh in such a way that the irregular surfaces are properly modeled. If the surfaces are not essentially coincident with the faces of the elements in the mesh (or more particularly some of the element faces,) then this advantage is minimized or lost. It would therefore be useful to develop a method for defining the elements of the unstructured mesh so that the surface is accurately defined by one or more faces of the elements of the mesh.

SUMMARY OF THE INVENTION

[8] One or more of the problems outlined above may be solved by the various embodiments of the invention. Broadly speaking, the invention comprises a method for adapting an unstructured finite element mesh to accurately model a surface in the region to be modeled. This method may provide a number of advantages over prior art methods, including increasing the accuracy of the model, simplifying the model, eliminating the need to use special functions to approximate the behavior of features such as well bores, and reducing the computing resources needed to solve the model.

[9] In one embodiment, the method comprises constructing an unstructured mesh corresponding to the modeled space, defining a surface in the modeled space, determining the intersections of the surface with the elements of the

4

mesh, and defining new elements within the intersected elements such that one or more of the faces of the new elements are substantially coincident with the surface (that is, they lie approximately on the surface.) More specifically, for each element which is intersected by the surface, a plurality of points at which the edges of the element are intersected is determined. (If the element is only intersected at a single point, it need not be subdivided into two or more new elements.) For each point of intersection, a new node is located at that point and two new elements which incorporate the new node is created. This process is performed for each of the points of intersection so that the intersected element is subdivided into an appropriate number of new elements, some of which include faces that lie substantially on the modeled surface.

[10] Various alternative embodiments are also possible. These may include software or firmware applications which embody methods such as are described above, as well as computer-readable media containing such applications and computers or other data processing systems which are configured to perform the described methods.

BRIEF DESCRIPTION OF THE DRAWINGS

[11] Other objects and advantages of the invention may become apparent upon reading the following detailed description and upon reference to the accompanying drawings in which:

[12] FIGURE 1 is an illustration of the rectangular and hexahedral elements which comprise structured finite element meshes in the prior art;

[13] FIGURE 2 is a flow diagram illustrating the basic steps
      of one embodiment of the present method;

[14] FIGURES 3A-3B are illustrations of a curve in two-
      dimensional space and an initial unstructured mesh that
      could be used as the basis for modeling the curve;

[15] FIGURES 4A-4C are a set of diagrams illustrating the
      manner in which an element of the unstructured mesh of
      FIGURE 3B is subdivided into sub-elements which can more
      accurately model the curve which intersects the original
      element;

[16] FIGURE 5 is an illustration of a refined mesh which is
      adapted to model the curve shown in FIGURES 3A and 3B;

[17] FIGURE 6 is a diagram illustrating a three-dimensional
      simplex in comparison with a two-dimensional simplex;

[18] FIGURE 7A-7D are a set of diagrams illustrating the
      possible types of intersections between an approximately
      planar surface and a three-dimensional simplex;

[19] FIGURES 8A-8C are a set of diagrams illustrating the
      manner in which an the three-dimensional simplex of
      FIGURE 7B is subdivided into sub-elements based upon its
      intersection by an approximately planar surface at one
      node and two edges;

[20] FIGURES 9A-9D are a set of diagrams illustrating the
      manner in which an the three-dimensional simplex of
      FIGURE 7C is subdivided into sub-elements based upon its
      intersection by an approximately planar surface at three
      edges;

[21] FIGURES 10A-10C are a set of diagrams illustrating the manner in which the three-dimensional simplex of FIGURE 7C is subdivided into two sub-elements based upon its intersection by an approximately planar surface at four edges;

[22] FIGURES 11A-11C are a set of diagrams illustrating the manner in which the first of the sub-elements of FIGURE 10C is further subdivided into sub-elements;

[23] FIGURES 12A-12C are a set of diagrams illustrating the manner in which the second of the sub-elements of FIGURE 10C is further subdivided into sub-elements; and

[24] FIGURE 13 is a diagram illustrating a well bore trajectory and the well bore surface defined by the trajectory.

[25] While the invention is subject to various modifications and alternative forms, specific embodiments thereof are shown by way of example in the drawings and the accompanying detailed description. It should be understood, however, that the drawings and detailed description are not intended to limit the invention to the particular embodiment which is described. This disclosure is instead intended to cover all modifications, equivalents and alternatives falling within the spirit and scope of the present invention as defined by the appended claims.

DETAILED DESCRIPTION OF A PREFERRED EMBODIMENT

[26] A preferred embodiment of the invention is described below. It should be noted that this and any other

embodiments described below are exemplary and are
intended to be illustrative of the invention rather than
limiting.

[27] The present method and system are directed generally to
constructing and solving finite element models. More
particularly, they are directed to the construction of
finite element meshes and the refinement of these meshes
so that the faces of some of the elements are coincident
with identified surfaces in the modeled space.

[28] Broadly speaking, the present method comprises
constructing an initial unstructured mesh of simplex
elements, identifying intersections between the elements
of the mesh and a surface to be modeled, then redefining
the intersected elements to include faces coincident with
the surface. More particularly, new nodes are located at
the points of intersection and are used as the basis for
subdividing the initial elements into new elements. The
number of new elements which are created from a
particular element depends upon the manner in which the
surface intersects that element. The new elements are
created in such a way that they are simplices in the
modeled space. The present method is employed in the
generation of multi-dimensional finite element models.
It should be noted that, while the examples described
below include only two- and three-dimensional meshes, the
method is generally applicable in n dimensions.

[29] It may be helpful before describing the present method in
detail to define a few terms relating to finite element
meshes.

[30] A node is a point in space. In finite element modeling, nodes form the vertices of the elements which are modeled. The nodes also form part of a mesh of nodes and edges which define the boundaries between elements in the modeled space.

[31] An edge is a line between two nodes which form vertices of an element. The edges form part of the mesh which defines the boundaries between elements in the modeled space.

[32] A simplex is a spatial configuration of n dimensions determined by n + 1 points in a space of dimension equal to or greater than n. In other words, a simplex is a geometric spatial element having the minimum number of boundary points necessary to enclose a space in a given number of dimensions. For example, in two dimensions, a simplex comprises a triangle, together with the interior area bounded by the triangle (see FIGURE 6.) Two points are insufficient to form a simplex in two-dimensional space because no area is bounded by the points (and the lines which interconnect them.) While four points may be sufficient to bound a two-dimensional area, they do not comprise the minimum number of boundary points by which the two-dimensional area can be bounded. In three dimensions, a simplex comprises a tetrahedron, together with the interior volume bounded by the tetrahedron. It is defined by four vertices (see FIGURE 6.) In four dimensions, a simplex is a hypertetrahedron (sometimes referred to as a hypertet), along with the four-dimensional volume inside the hypertetrahedron, and has five vertices.

[33] A face is an n-1-dimensional surface of an n-dimensional element. For example, in a two-dimensional element, each of the lines which bounds the area of the element is a face of that element. In a three-dimensional element, each of the triangles which bounds the volume of the element is a face of that element. Preferably, the faces of the elements in a finite element model are located so that they are coincident with surfaces or boundaries within the modeled space.

[34] A mesh is a collection of elements that fill a modeled space. These elements are representative of a system which resides in that space. Because each element can be defined by a plurality of nodes and/or the edges between those nodes, a mesh may alternatively be considered a collection of nodes and/or the edges between them. At various points in this disclosure, "mesh" will be used to alternately refer to collections of elements or nodes/edges, depending upon the context in which the term is used. The mesh may also be referred to herein as a finite element model or simply a model.

[35] Referring to FIGURE 2, a flow diagram illustrating the basic steps of one embodiment of the present method is shown. Generally speaking, the method comprises constructing an unstructured mesh corresponding to a space which is being modeled, then identifying a surface in the modeled space and determining the intersections of the surface with the elements of the mesh, and finally defining new elements within the intersected elements so that one or more of the faces of the new elements are substantially coincident with the surface. ("Substantially coincident" is used herein to refer to

element edges or faces for which the corresponding nodes
lie on a curve or surface.) These faces approximate the
modeled surface.

[36] Each element in the mesh has a corresponding set of nodes
which comprise the vertices of the element. Likewise,
each element has a corresponding set of edges which
connect the vertices. In the present method, the
intersection of a surface with an element is
characterized by the points at which the surface
intersects with the nodes and/or edges of the element.
Consequently, identifying the intersection of a surface
with the element comprises determining the points at
which the surface intersects the nodes and/or edges.
These points of intersection are then used as nodes for
two or more newly defined elements. The new elements
effectively include the intersecting portion of the
surface as one of the faces of the new elements.

[37] A two-dimensional example is illustrated in FIGURES 3-4.
FIGURE 3A shows a curve 11 which represents a surface in
two-dimensional space. FIGURE 3B shows a two-dimensional
mesh which is constructed to model the space containing
curve 11. The mesh comprises a series of triangular
elements which are simplices in two-dimensional space.
One of the elements, identified by reference number 12,
is highlighted -- this element will be referred to in the
following description of the manner in which the elements
are subdivided. Although the elements in this initial
mesh are regular, it should be noted that the mesh is
unstructured, and the elements may have any suitable size

or shape (keeping in mind that they must be simplices.)
Curve 11 is also shown in this figure.

[38] It can be seen that the elements of the initial mesh do
not follow the curve. In other words, the curve does not
lie substantially along the edges of the elements.
Instead, the curve intersects a number of the elements,
traversing some path across the space bounded by these
elements. Because it is desired to construct a mesh in
which the faces of the elements coincide with the curve,
it is necessary to modify or refine the mesh. In the
present method, this is accomplished by effectively
adding edges that lie on the curve (subdividing the
existing elements into new elements which include the
added edges.)

[39] Referring to FIGURES 4A-4C, element 12 is shown apart
from the mesh. A portion of curve 11 which intersects
element 12 is also shown in this figure. Element 12 is
defined by the three points (21, 22, 23) which form its
vertices. The area within element 12 is bounded by the
edges (31, 32, 33) which connect the vertices. As
indicated above, it is necessary to identify the
intersections between curve 11 and edges 31, 32 and 33.
It is a relatively straightforward matter to calculate
the intersections between the curves and the edges based
upon the mathematical representation of each. It can be
seen from FIGURE 4A that curve 11 intersects edge 31 at
point 24. Likewise, curve 11 intersects edge 32 at point
25.

[40] An edge connecting points 24 and 25 would be
substantially coincident with curve 11 in the space

covered by element 12. That is, an edge connecting
points 24 and 25 would closely approximate the portion of
curve 11 which traverses element 12. Creation of an edge
between points 24 and 25, however, would not, by itself,
be a sufficient refinement of the mesh because this would
divide element 12 into a triangular area and a second
area bounded by an irregular quadrilateral, which is not
a simplex in two-dimensional space. The space covered by
the quadrilateral must therefore be subdivided into
simplices (triangular elements.)

[41] Referring to FIGURES 4B and 4C, the manner in which
element 12 is subdivided into a plurality of new simplex
elements is illustrated. Once the locations of points 24
and 25 are identified, element 12 is subdivided as
follows: first, a node is placed at point 24 and an edge
is created between this node and the opposite vertex of
element 12 (point 23.) ("Opposite," as used here, refers
to the vertex which is opposite the edge (31) on which
the new node is located.) This creates new elements 13
and 14. Element 13 is defined by vertices at points 21,
23 and 24, while element 14 is defined by vertices at
points 22, 23 and 24. Then, a node is placed at point 25
and an edge is created between this node and the node at
point 24 (the opposite vertex of element 14.) This edge
forms a segment of the modeled curve. As shown in FIGURE
4C, the addition of this edge results in the creation of
new elements 15 and 16. Element 15 is defined by
vertices at points 23, 24 and 25, while element 16 is
defined by vertices at points 22, 24 and 25.

[42] Now, instead of having a single simplex element 12 which
is intersected by curve 11 (and does not have a face

13

which is coincident with the curve,) there are three new
elements (13, 15 and 16,) each of which is still a
simplex in two-dimensional space, and two of which share
a face that lies on curve 11. This same process can be
repeated with each of the original mesh elements so that
an edge is created wherever curve 11 traverses one of the
elements. The result is an unstructured mesh of simplex
elements which have edges that closely approximate the
path of curve 11 and consequently enable the accurate
modeling of the surface represented by the curve.

[43] Referring to FIGURE 5, the refined mesh generated by
applying the foregoing method to each of the intersected
elements of FIGURE 3B is shown. The edges of the new
simplices which are substantially coincident with curve
11 are shown in heavy lines so they can be easily seen.
It is apparent from the figure that the new edges provide
a close approximation of the original curve. The curve
may be matched even more closely if the mesh is refined
to use smaller elements before the present method is
applied, since smaller edges do a better job of
approximating curved surfaces.

[44] While each of the elements of the mesh illustrated in
FIGURE 3B were intersected by curve 11 from one edge to
another, it should be noted that the elements could also
have been intersected at a node and an edge. That is,
for a particular element, curve 11 could have traversed
the element between one of its nodes and the opposite
edge. In this case, it would only be necessary to make a
single subdivision of the element by creating a new node
where the curve intersects the edge, and then defining
each of the new elements to include the intersected node,

the new node, and one of the remaining nodes. For
example, in the example of FIGURES 4A and 4B, if the
curve had traversed the element from point 24 to point
23, a new edge between these points (and resulting
elements 13 and 14) would have been sufficient.

[45] There are actually two other cases which may be
      encountered. In one case, the surface may intersect only
      a single node of the simplex element. Because this node
      lies on the surface, there is no need to refine the
      element further. The surface may also intersect the
      element at two nodes. In this instance, the edge of the
      element already lies on the surface to be modeled, so no
      refinement of the element is necessary. (It should be
      noted that the identification of the type of intersection
      between the element and the surface may be regarded as a
      separate step of the method.)

[46] The foregoing example is a very simple illustration of
      one embodiment of the present method in the context of a
      two-dimensional mesh. The more complex case of a three-
      dimensional mesh is illustrated below.

[47] Meshes of three and four dimensions are of particular
      interest in modeling physical systems over time. Because
      of the difficulty of illustrating four-dimensional
      systems, only the three-dimensional case will be
      described in detail below. The description of the three-
      dimensional case illustrates the manner in which the
      method can be extended to additional dimensions.

[48] Referring to FIGURE 6, a three-dimensional simplex 41 is
      shown in comparison to a two-dimensional simplex 40.
      three-dimensional simplex 41 is defined by four vertices

42-45 and is bounded by the edges and faces between them. The application of the present method is, of course, somewhat more complicated in the three-dimensional case than in the two-dimensional case. One added complexity arises from the need to calculate the intersection of a three-dimensional surface with a three-dimensional edge. While this requires additional computing power with respect to the two-dimensional case, it is still relatively straightforward. Another complexity is that the intersection of a surface and a simplex may involve four different cases which could necessitate the subdividing of the simplex: intersecting one node and two edges; intersecting two nodes and one edge; intersecting three edges; or intersecting four edges. Each of these cases is handled differently. (Remember that a curve in two dimensions could also intersect a simplex at one, two or three nodes, although no subdivision of the simplex would be required in these instances.)

[49] Although more complex, the method applied to three-dimensional meshes is generally the same as in the two-dimensional case – an initial mesh is constructed, a surface is identified, the intersections between the surface and the edges of the elements are identified, and the elements are subdivided using the intersections as nodes in the new elements. The three-dimensional method differs in that the step of subdividing the element is handled a bit differently for each of the four different types of intersections. (It should be noted that the present methods can be extended to four or more dimensions, but such cases will not be described here because of the difficulty of illustrating them clearly.)

[50] The four different types of intersections in three
dimensions are illustrated in FIGURES 7A-7D. Each figure
depicts the three-dimensional simplex element of FIGURE 6
intersected by a surface having a different orientation.
The shaded portion of the figure represents the portion
of the surface that intersects the element. FIGURE 7A
shows the intersection of the element at two nodes (44
and 45) and one edge (51.) FIGURE 7B shows the
intersection of the element at one node (43) and two
edges (52 and 53.) FIGURE 7C shows the intersection of
the element at three edges (54 - 56.) Finally, FIGURE 7D
shows the intersection of the element at four edges (57-
60.)

[51] (It should be noted that the surface may also intersect
the three-dimensional element at one, two or three nodes.
In the first two of these cases, the surface does not
traverse the element and no subdivision of the element is
necessary. In the third case, the face of the element
already lies on the surface and no further action needs
to be taken.)

[52] Referring to the intersection depicted in FIGURE 7A,
where the surface intersects the element at two nodes and
one edge, the element is subdivided by creating a face
between the two nodes (44 and 45) and the point of
intersection on the edge (51). This results in the
creation of two tetrahedra with the vertices {43, 44, 45,
51} and {42, 44, 45, 51}. Newly created face {44, 45,
51} is a facet of the surface which is being modeled.

[53] Referring to the intersection depicted in FIGURE 7B, the
surface intersects the element at one node and two edges.

This figure is repeated as FIGURE 8A for the convenience
of the reader. The points of intersection with the edges
of element 41 are indicated by reference numerals 71 and
72. The first step in subdividing the element is to
select one of the points of intersection and to locate a
node at that point. In this example, point 71 is
selected. As shown in FIGURE 8C, two new simplex
elements, indicated by reference numbers 75 and 76, can
be created using this new node. The first new element,
75, has vertices at {42, 43, 45, 71} and the second, 76,
has vertices at {43, 44, 45, 71}. The next step is to
place a node at the remaining point of intersection (72).
This node is used as the basis for subdividing element 75
into new elements 77 and 78 (see FIGURE 8C.) Element 77
has vertices at {42, 43, 71, 72} and element 78 has
vertices at {43, 45, 71, 72}. Thus, the subdivision of
element 41 in this case results in the creation of new
elements 76, 77 and 78. The face which separates
elements 77 and 78 forms a facet which is substantially
coincident with the modeled surface.

[54] Referring to the intersection depicted in FIGURE 7C, the
surface intersects three edges of element 41. This
figure is repeated as FIGURE 9A for the convenience of
the reader. The points of intersection between the
surface and element 41 are indicated by reference
numerals 81, 82 and 83. The process of subdividing
element 41 in this case is very similar to the previous
case -- nodes are sequentially located at the points of
intersection and new elements are created based on the
addition of each node. For example, as shown in FIGURE
9B, a first node is located at point 81, creating

elements 84 and 85. Element 84 has vertices at {43, 44,
45, 81} and element 85 has vertices at {42, 44, 45, 81}.
Then, a node is placed at the next point of intersection
(82). This node is used to subdivide element 85,
creating new elements 86 and 87. Element 86 has vertices
at {44, 45, 81, 82} and element 87 has vertices at {42,
44, 81, 82}. Finally, a node is placed at the last point
of intersection (83). Using this node, element 87 is
subdivided into new elements 88 and 89. Element 88 has
vertices at {44, 81, 82, 83} and element 89 has vertices
at {42, 81, 82, 83}. Thus, in this case, element 41 is
subdivided into four new elements -- 84, 86, 88 and 89.
The face which separates elements 88 and 89 forms a facet
of the intersecting surface and is substantially
coincident with the surface.

[55] Referring to the intersection depicted in FIGURE 7D, the
surface intersects four edges of element 41. This figure
is repeated as FIGURE 10A for the convenience of the
reader. The points of intersection between the surface
and element 41 are indicated by reference numerals 91, 92
93 and 94. The subdivision of element 41 is illustrated
in FIGURES 10A-10C, 11A-11C and 12A-12C. It should be
noted that the first step in subdividing element 41
creates two new elements, 95 and 96, each of which is
intersected by the surface at one node and two edges.
This is illustrated in FIGURES 10B-10C. Each of these
new elements can then be subdivided in the manner
illustrated in FIGURES 7B-7C. The subdivision of
elements 95 and 96 is shown in FIGURES 11A-11C and 12A-
12C.

[56] Referring to FIGURES 11A-11C, the subdivision of element
95 is illustrated. First, a node is placed at point 92,
and two new elements incorporating this node are created.
These are elements 97 and 98. The vertices of element 97
are at {42, 43, 91, 92} and the vertices of element 98
are at {42, 45, 91, 92}. Then, a node is placed at point
93. Based on this new node, element 98 is broken down
into two new elements, 99 and 100. Element 99 has
vertices at {42, 91, 92, 93} and element 100 has vertices
at {45, 91, 92, 93}. The face of elements 99 and 100
which is formed between points 91, 92 and 93 forms a
facet of the modeled surface.

[57] Referring to FIGURES 12A-12C, the subdivision of element
96 is illustrated. The first step is to place a node at
point 93. Based on this node, two new elements are
created. These elements are indicated by the reference
numerals 101 and 102. The vertices of element 101 are at
{42, 44, 91, 93} and the vertices of element 102 are at
{44, 45, 91, 93}. Next, a node is placed at point 94.
Using this new node, element 101 is broken down into two
new elements, 103 and 104. Element 103 has vertices at
{42, 91, 93, 94} and element 104 has vertices at {44, 91,
93, 94}. The common face of elements 103 and 104 which
lies between points 91, 92 and 93 forms another facet of
the modeled surface.

[58] One application of the adaptive meshing methods described
above involves the modeling of a well bore that follows
an arbitrary path through a reservoir. In prior art
systems, well bores were typically assumed to be aligned
with the structured mesh which was used to model the
reservoir. Then, functions approximating the behavior of

the reservoir in the vicinity of the well bore were used
for the elements containing the well bore when the model
was solved. Generally speaking, these functions do not
model the well bore as accurately as if the well bore
surface were represented by the faces of the mesh
elements, and were not at all well suited to modeling
well bores having arbitrary paths.

[59] Using the present method, a well bore in a reservoir can
be accurately modeled by refining the elements of the
finite element mesh to define element faces which are
substantially coincident with the well bore surface.
This is true even if the well bore follows an arbitrary
path through the reservoir. The well bore surface is
simply defined mathematically, and then the mesh is
refined as described above.

[60] In one embodiment, the well bore is defined by a
trajectory and a radius. The trajectory is the path
which is followed by the well bore and consists of a set
of points that are ordered by their respective measured
depths within the well bore. The radius is the actual
radius of the well bore. The well bore can therefore be
characterized mathematically by a function $F(x,y,z)-$
$r(z)=0$, where $F(x,y,z)$ is the distance from a point in
the reservoir to the trajectory of the well bore and r is
the radius of the well bore. For each of the edges in
the finite element mesh (or a portion of the edges that
are within the vicinity of the well bore) intersections
with the defined well bore surface are identified. The
points at which the edges are intersected by the well
bore surface are marked for refinement. The elements
containing the marked edges are then refined as described

above. It should be noted that the elements for which
four edges are intersected are refined first. In order
to maintain surface preservation, no adjacent elements
having four intersected edges are refined in a single
pass. If such adjacent elements are identified, one is
identified for deferral to a subsequent refinement pass.
It may therefore take several passes to complete the
refinement of these elements. Once the elements having
four intersected edges are refined, the remainder of the
intersected elements can be refined in a single pass.

[61] Because the path of the well bore through the reservoir
is arbitrary, it is necessary to define a coordinate
system which is convenient for both defining the well
bore and identifying the intersection of the well bore
with the elements of the mesh. In one embodiment, a
coordinate system which tracks the path of the well bore
is used. This coordinate system identifies points around
the well bore using the measured depth in the well, an
angular distance from a normal vector and a radial
distance from the center of the well bore.

[62] The well path is assumed to comprise a series of
segments, each of which is essentially straight.
Adjacent segments are joined at joints to form a
continuous well path. Because adjacent segments are not
coaxial, the coordinate system must account for the fact
that some points near the joints may not be well defined.
(That is, the coordinate system defined with respect to
adjacent segments may overlap on one side of the joint
and may not cover all points on the opposite side of the
joint.) Consequently, a region corresponding to each
segment is defined so that the region extends to a

bisecting plane between that segment and an adjacent
segment. This eliminates overlapped or undefined areas.
(It should be noted that, for a segment at the end of the
well bore, the end of the segment region near to an
adjacent segment is bounded by the bisecting plane
between the segments, but the other end of the segment
region is bounded by a plane normal to the trajectory of
the segment.)

[63] As indicated above, the coordinate system for each
segment includes a measured depth, an angular distance
from a normal vector and a radial distance from the path
of the well bore. The measured depth of points on the
trajectory itself are well-defined. The measured depth
of points which are on the inside or outside of curves in
the trajectory, however, are not so well-defined.
Consequently, the following method for determining the
measured depth of these points is imposed on the system.

[64] Referring to FIGURE 13, a diagram illustrating a well
bore trajectory 110 and the well bore surface 111 defined
by the trajectory is shown. A segment trajectory 114 has
a beginning depth D_b and an ending depth D_e. For a given
point P in the well bore, the point is first projected
onto the bisecting (or terminating) planes 116, 117 at
the ends of the segment. The projection at the beginning
of the segment can be indicated by P_b and the projection
at the end of the segment can be indicated by P_e. The
distance between these two projections of the point is
then mapped onto the distance between the beginning and
ending depths of the trajectory. In other words, the
measured depth at point P (D_p) can be calculated as $D_p =
D_b + (P-P_b) \times (D_e-D_b)/(P_e-P_b)$. Thus, effectively, the

depth along the well bore trajectory is "stretched" along the outside of curves in the trajectory and "compressed" along the inside of curves in the trajectory.

[65] It should also be noted that the normal vector for a segment may become ill-defined if it is not translated from one segment to another. For example, the trajectory of the well bore may curve so that the trajectory is parallel to the original normal vector. Consequently, the normal vector for each segment of the trajectory is defined as follows. First, at the beginning of the well bore, a normal vector 120 is defined. This vector extends in an arbitrary direction from the well bore trajectory. The normal vector is then projected onto the bisecting plane 117 at the end of the initial segment of the well bore. The projected vector 121 is then carried through to the next segment of the trajectory and made normal to the trajectory in that segment. This process is repeated from segment to segment throughout the well bore. The angular coordinate A of a point in any of the segments can then be measured from the normal vector in the respective segment.

[66] The radial distance from the well bore path is calculated in a straightforward manner -- it is simply the closest distance from the trajectory of the segment.

[67] In one embodiment, each node near the well bore is converted to the well bore coordinate system, and the converted coordinates are stored with the node. Thus, the coordinate transformation is performed only once, and the coordinates of each node are available in the original and transformed coordinate systems.

[68]   In the foregoing manner, a well bore having an arbitrary
       path defined by the trajectory F(x,y,z) and radius r can
       be characterized in a convenient mathematical form. The
       nodes of the finite element mesh in the vicinity of the
       well bore can also be characterized using this same local
       coordinate system. This facilitates identification of
       the intersections between the well bore surface and the
       elements of the mesh. Once the points of intersection
       between the elements' edges and the well bore surface are
       identified, the elements are refined as described above.
       In this manner, an arbitrary-path well bore can be
       accurately modeled in an unstructured finite element
       mesh.

[69]   Another application of the adaptive meshing methods
       described above involves the refinement of a finite
       element mesh to increase the accuracy of a solution for
       the mesh. In this application, the mesh may not be
       intersected by a particular plane which defines the
       points at which the finite element mesh needs to be
       refined. Instead, a general area in which refinement is
       required may be identified, and elements within this area
       may be appropriately subdivided.

[70]   One embodiment of this application comprises a method
       wherein a finite element mesh made up of simplex elements
       is constructed, and a solution is calculated for the
       mesh. Portions of the mesh for which the solution has
       large gradients (i.e., those for which the solution
       changes rapidly) are then identified. Simplices in the
       high-gradient portions of the mesh are marked for
       refinement. This marking may comprise the identification
       of simplex edges, defining new nodes on these edges

(typically around the midpoint of each edge,) and
subdividing the corresponding simplex elements as though
the elements were intersected at the point of the new
node and the opposite edge. After the elements are
subdivided (i.e. the mesh is refined,) a new solution is
calculated for the finite element mesh. This process may
be repeated as necessary to obtain the desired level of
accuracy (number of data points) in the high-gradient
areas. The addition of simplices only in the high-
gradient areas increases the accuracy of the solution
without unreasonably increasing the computational
requirements of the mesh (as would be the case if the
entire mesh were made more fine.)

[71] It should be noted that, just as the simplex elements of
the mesh can be refined to include additional nodes in
high-gradient areas, the mesh can be modified to remove
unnecessary nodes in low-gradient areas. The simplex
elements which are associated with the removed nodes are
collapsed into a smaller number of elements. This
reduces the processing power required to generate a
solution for the mesh. The computational requirements of
the application can also be reduced by implementing
table-driven algorithms to collapse the simplices into a
smaller number of elements. (The use of table-driven
algorithms also reduces the computational complexity in
the case of refining a simplex into a plurality of
elements.) The refinement or coarsening of simplices may
be limited to preselected regions of the mesh, if it can
be determined that certain areas need not be modified
(e.g., they are not of particular interest.)

[72] In an application which is configured to collapse
     simplices within the mesh, it may be desirable to employ
     a mechanism to prevent certain nodes from being
     eliminated. For example, certain nodes may correspond to
     the features of the topology of the modeled system (e.g.,
     known geological surface features within an oil
     reservoir.) In order to maintain the topology of the
     modeled system, the corresponding nodes may be designated
     as invariant, so that they will not be removed or
     "optimized away."

[73] While the present invention has been described with
reference to particular embodiments, it should be understood
that the embodiments are illustrative and that the scope of
the invention is not limited to these embodiments. Many
variations, modifications, additions and improvements to the
embodiments described above are possible. Particularly, these
variations may include computer readable media (such as floppy
disks, CD-ROMs, DVD-ROMs, etc.,) storage devices, computer
memories and the like which contain software, firmware or
other programming embodying the foregoing methods. It is
contemplated that these variations, modifications, additions
and improvements fall within the scope of the invention as
detailed within the following claims.

WHAT IS CLAIMED IS:

1.   A method for modifying the structure of an n-dimensional
finite element mesh comprising:

      constructing a mesh having a plurality of n-dimensional
          simplices;

      identifying a subset of the plurality of n-dimensional
          simplices; and

      modifying the identified subset of the plurality of n-
          dimensional simplices to adapt the mesh to a modeled
          system.

2.   The method of claim 1 wherein modifying the identified
subset of the plurality of n-dimensional simplices comprises
subdividing each of the simplices in the identified subset
into a plurality of new simplices.

3.   The method of claim 1 wherein modifying the identified
subset of the plurality of n-dimensional simplices comprises
collapsing each of the simplices in the identified subset .

4.   The method of claim 3 wherein each of the simplices has a
plurality of nodes and a plurality of edges connecting the
nodes, wherein at least one of the nodes is invariant and
wherein collapsing each of the simplices in the identified
subset comprises removing one or more of the nodes, preventing
removal of the invariant nodes, and forming simplices based
upon the remaining nodes.

5.   The method of claim 1 wherein the modeled system
comprises a surface of dimension n-1 and wherein the subset of
the plurality of n-dimensional simplices is identified by the

intersection of the n-1-dimensional surface with the
identified subset of the plurality of n-dimensional simplices.

6.   The method of claim 5 wherein the n-1-dimensional surface
corresponds to a well bore, and wherein the n-1-dimensional
surface is defined by a depth along a well bore trajectory and
radius from the well bore trajectory.

7.   The method of claim 1 wherein the subset of the plurality
of n-dimensional simplices is identified by solving the
unmodified mesh, identifying a region having a high gradient,
and identifying a plurality of n-dimensional simplices in the
region having the high gradient.

8.   The method of claim 7 wherein the subset of the plurality
of n-dimensional simplices is selected from a predetermined
region within the mesh.

9.   The method of claim 7 wherein the steps of solving the
mesh, identifying the region having a high gradient, and
identifying the plurality of n-dimensional simplices in the
region having the high gradient are repeated one or more
times.

10.   The method of claim 1 wherein n=3.

11.   The method of claim 10 wherein each of the n-dimensional
simplices in the identified subset of the plurality of n-
dimensional simplices is intersected by a 2-dimensional
surface.

12. The method of claim 11 wherein the two-dimensional surface intersects at least one edge of each of the n-dimensional simplices in the identified subset of the plurality of n-dimensional simplices.

13. The method of claim 1 wherein n=4.

14. The method of claim 13 wherein the four dimensions of the mesh correspond to three spatial dimensions and one temporal dimension.

15. The method of claim 1 wherein each of the simplices has a plurality of nodes and a plurality of edges connecting the nodes and wherein the method further comprises providing a definition of a surface within a region modeled by the mesh, identifying intersections between the edges of the simplices and the surface, defining a new node at each of the identified intersections, and defining at least two new simplices incorporating the new nodes.

16. A method for modifying the structure of an n-dimensional finite element mesh comprising:
       constructing a mesh having a plurality of n-dimensional
             simplices corresponding to a modeled system;
       identifying a subset of the plurality of n-dimensional
             simplices which are intersected by a well bore
             surface contained in the modeled system; and
       refining the identified subset of the plurality of n-
             dimensional simplices to cause the well bore surface
             to be substantially coincident with the a plurality
             of faces of the identified subset of the plurality
             of n-dimensional simplices.

17.   The method of claim 16 wherein refining the identified
subset of the plurality of n-dimensional simplices comprises
subdividing each of the simplices in the identified subset
into a plurality of new simplices.

18.   The method of claim 16 wherein the well bore surface is
defined by a depth along a well bore trajectory and radius
from the well bore trajectory.

19.   A computer-readable medium containing a plurality of
instructions embodying the method comprising:
       constructing a mesh having a plurality of n-dimensional
            simplices;
       identifying a subset of the plurality of n-dimensional
            simplices; and
     modifying the identified subset of the plurality of n-
            dimensional simplices to adapt the mesh to a modeled
            system.

20.   The computer-readable medium of claim 19 wherein
modifying the identified subset of the plurality of n-
dimensional simplices comprises subdividing each of the
simplices in the identified subset into a plurality of new
simplices.

21.   The computer-readable medium of claim 19 wherein
modifying the identified subset of the plurality of n-
dimensional simplices comprises collapsing each of the
simplices in the identified subset .

22. The computer-readable medium of claim 21 wherein a portion of the mesh is held invariant during the collapsing of the simplices in the identified subset.

23. The computer-readable medium of claim 19 wherein the modeled system comprises a surface of dimension n-1 and wherein the subset of the plurality of n-dimensional simplices is identified by the intersection of the n-1-dimensional surface with the identified subset of the plurality of n-dimensional simplices.

24. The computer-readable medium of claim 23 wherein the n-1-dimensional surface corresponds to a well bore, and wherein the n-1-dimensional surface is defined by a depth along a well bore trajectory and radius from the well bore trajectory.

25. The computer-readable medium of claim 19 wherein the subset of the plurality of n-dimensional simplices is identified by solving the unmodified mesh, identifying a region having a high gradient, and identifying a plurality of n-dimensional simplices in the region having the high gradient.

26. The computer-readable medium of claim 25 wherein the subset of the plurality of n-dimensional simplices is selected from a predetermined region within the mesh.

27. The computer-readable medium of claim 25 wherein the steps of solving the mesh, identifying the region having a high gradient, and identifying the plurality of n-dimensional simplices in the region having the high gradient are repeated one or more times.

28.   The computer-readable medium of claim 19 wherein n=3.

29.   The computer-readable medium of claim 28 wherein each of
the n-dimensional simplices in the identified subset of the
plurality of n-dimensional simplices is intersected by a 2-
dimensional surface.

30.   The computer-readable medium of claim 19 wherein n=4.

OBJ1115-WO
1 OF 11

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┌─────────────────────┐
│   Construct finite   │
│    element mesh      │
└─────────────────────┘
           │
           ▼
┌─────────────────────┐
│  Identify surface in │
│    modeled space     │
└─────────────────────┘
           │
           ▼
┌─────────────────────┐
│ Identify intersections│
│  between surface and │
│    mesh elements     │
└─────────────────────┘
           │
           ▼
┌─────────────────────┐
│      Subdivide       │
│ intersected elements │
│   into new elements  │
└─────────────────────┘
```

Fig. 2

Y

X

11

Fig. 3A

Y

12

11

X
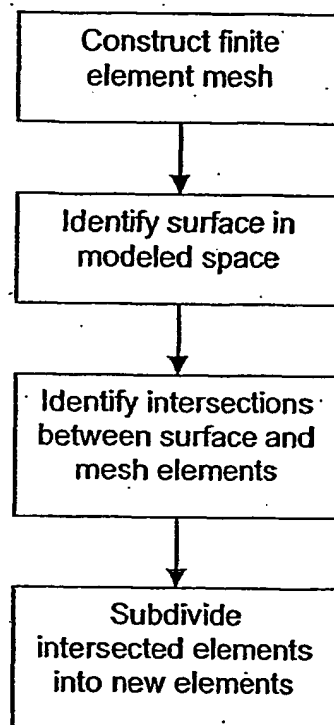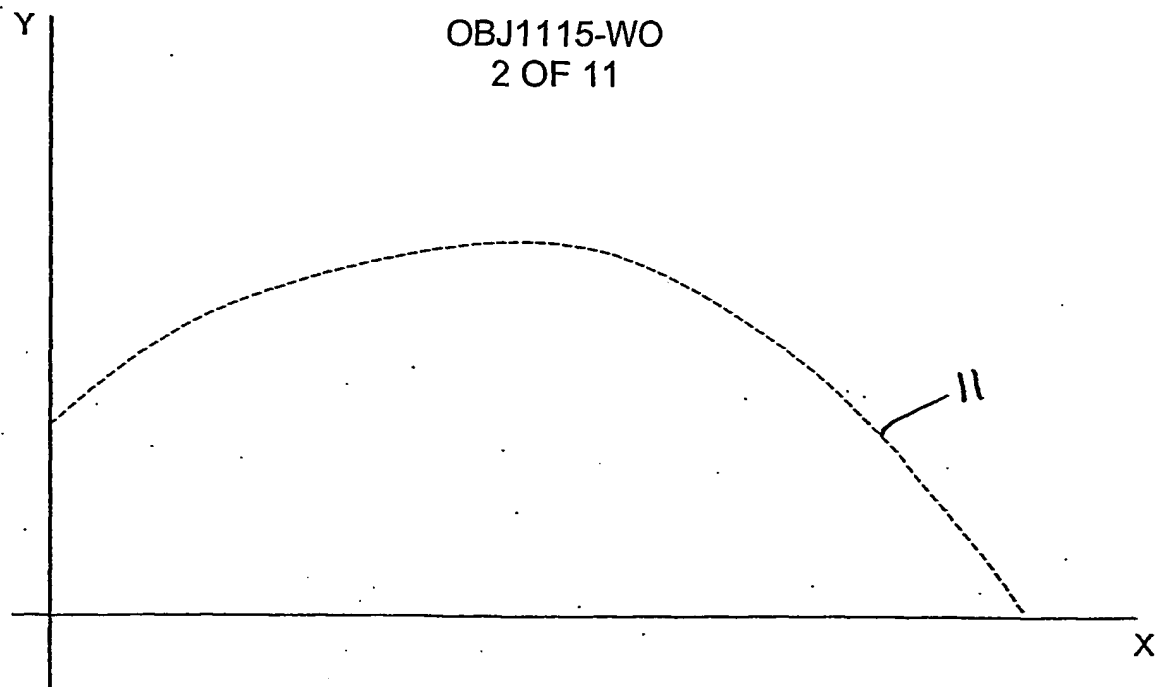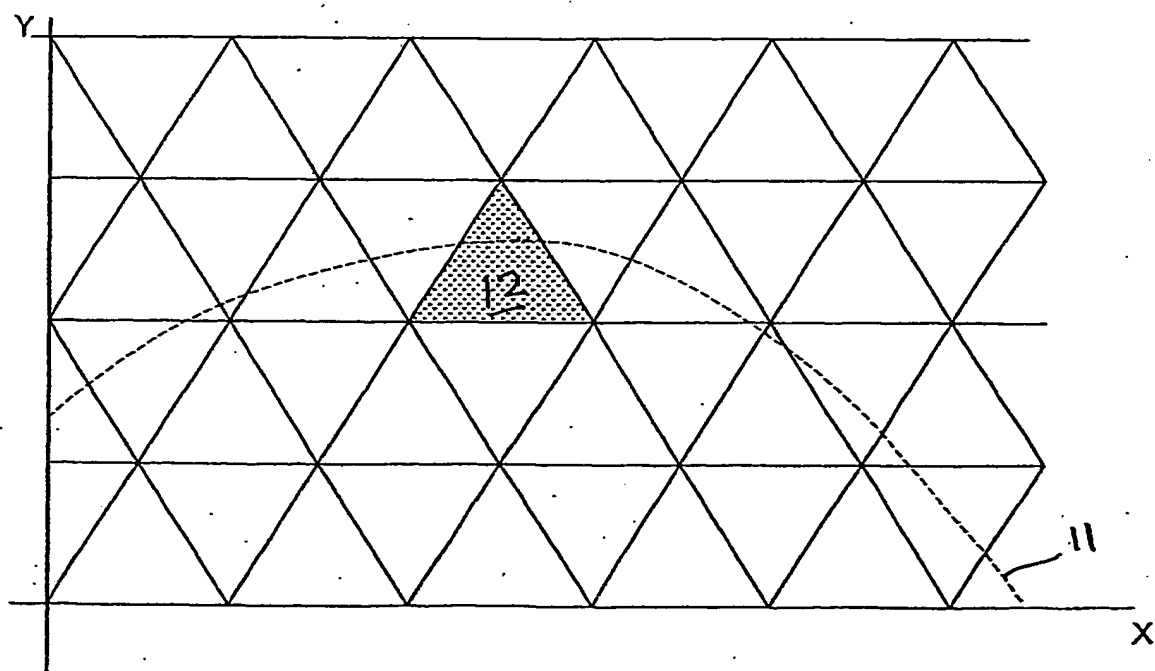
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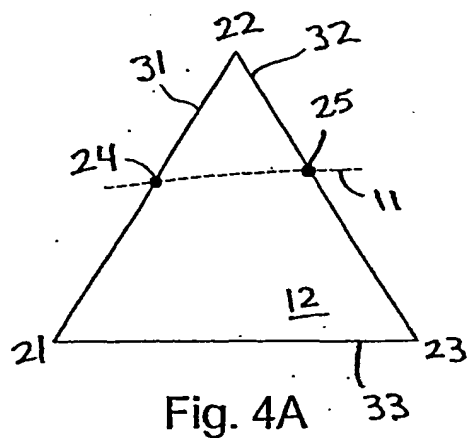
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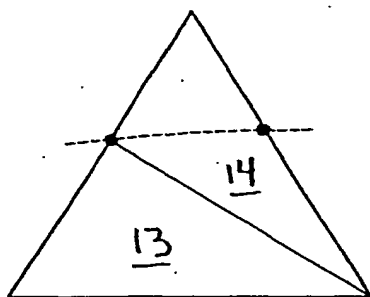
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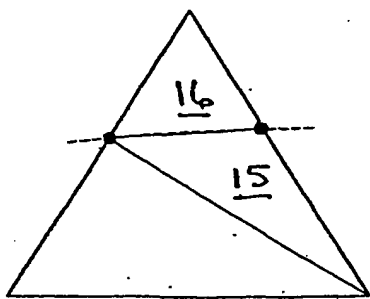

Fig. 4C

OBJ1115-WO
4 OF 11
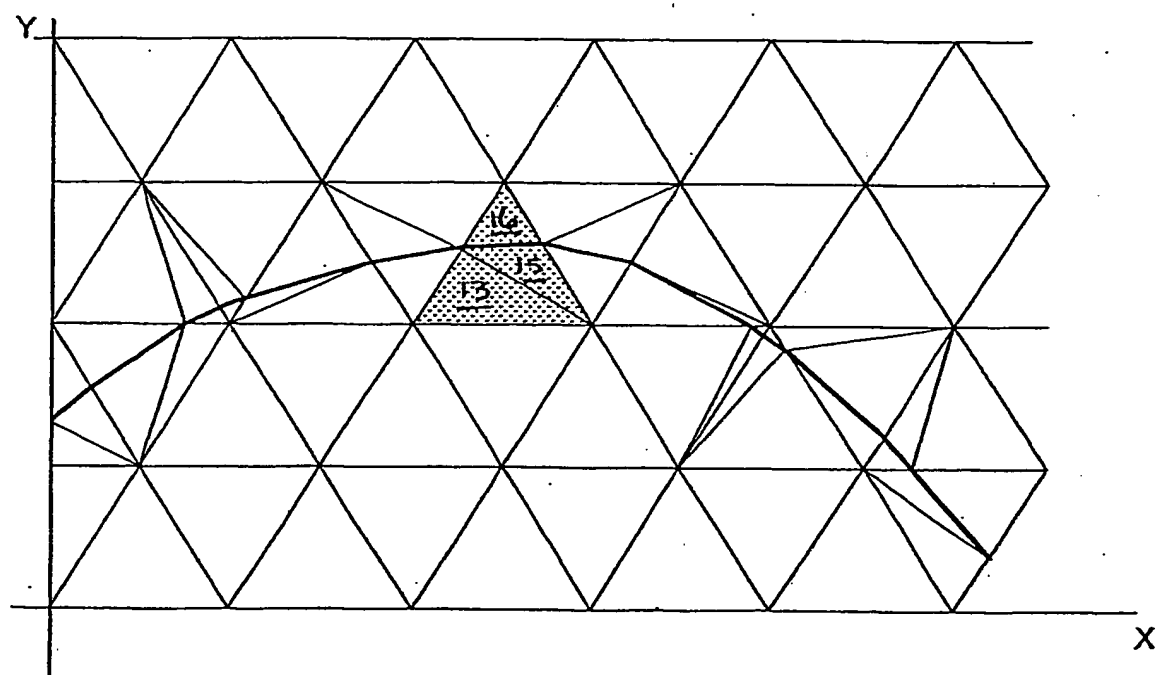
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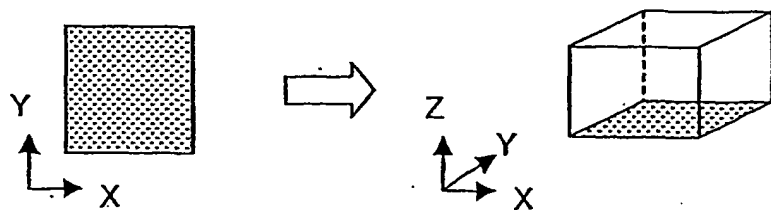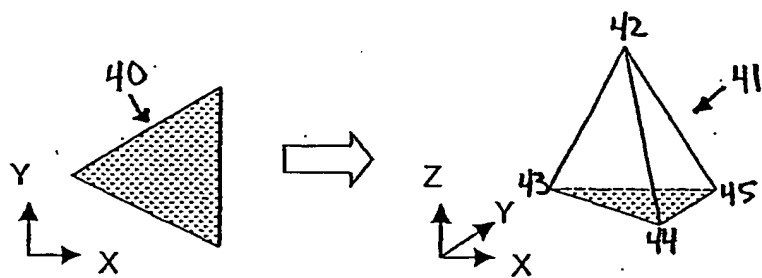
Fig. 1

Fig. 6
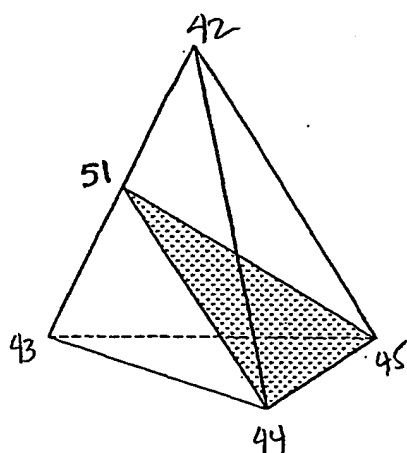
2 nodes, 1 edge

Fig. 7A
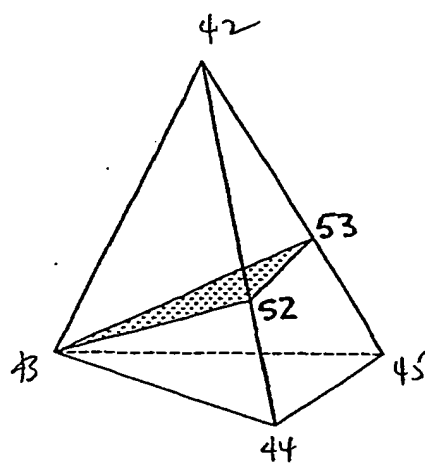
1 node, 2 edges

Fig. 7B
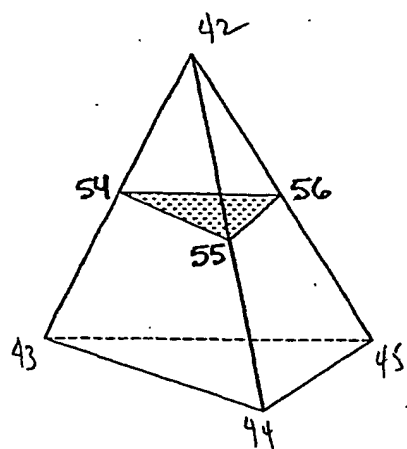
3 edges

Fig. 7C
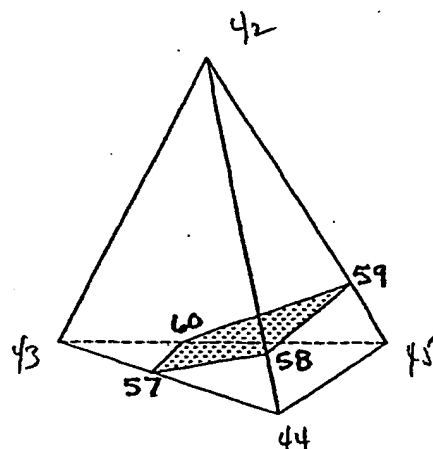
4 edges

Fig. 7D
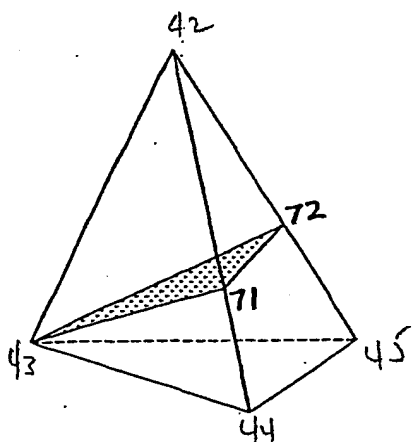
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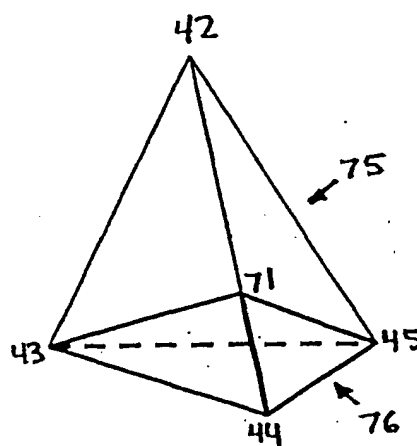

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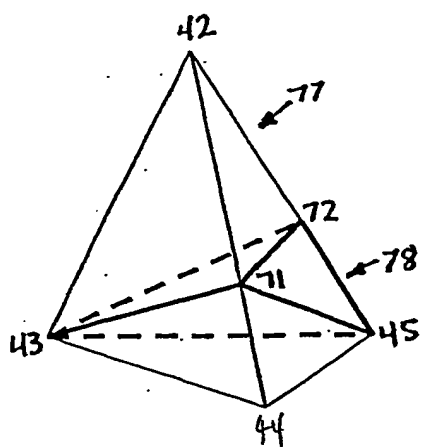
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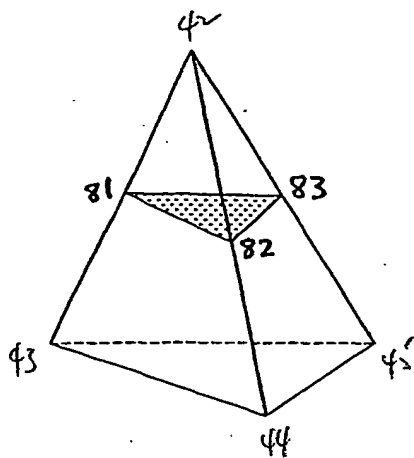
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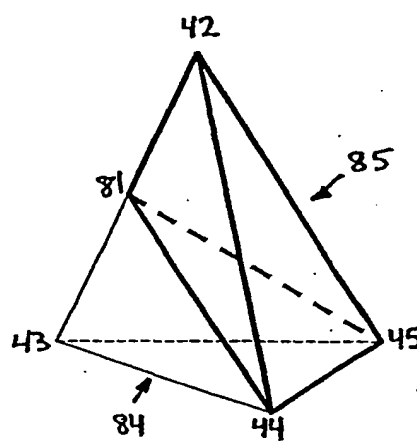
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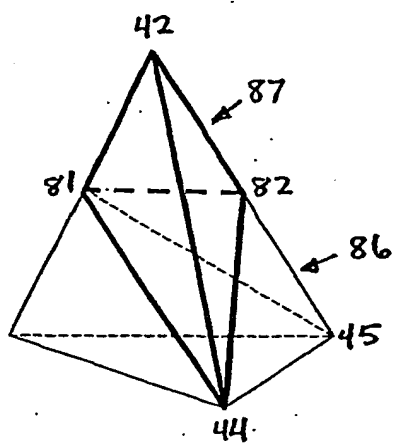
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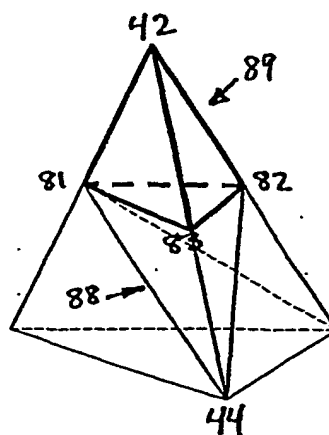
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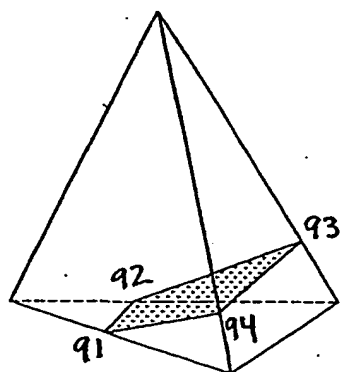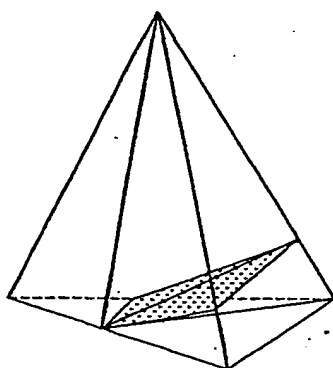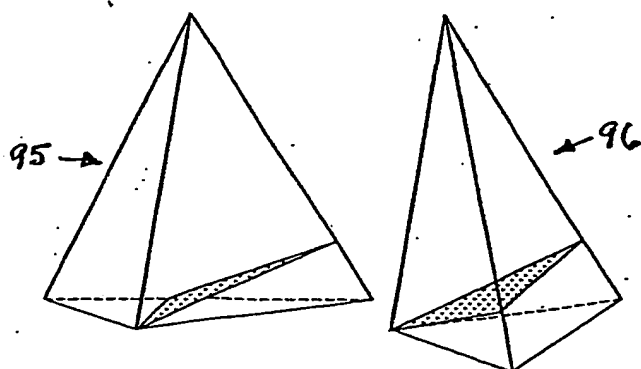
Fig. 10A

Fig. 10B

Fig. 10C
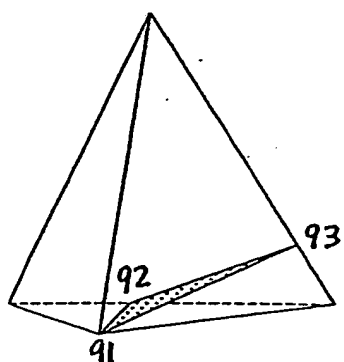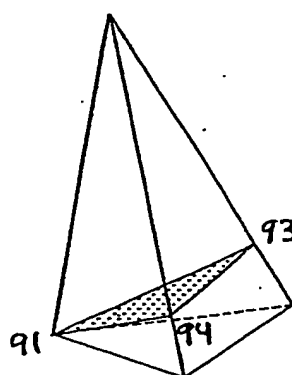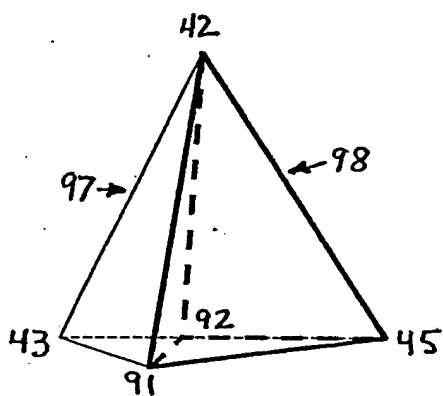
Fig. 11A

Fig. 12A

Fig. 11B
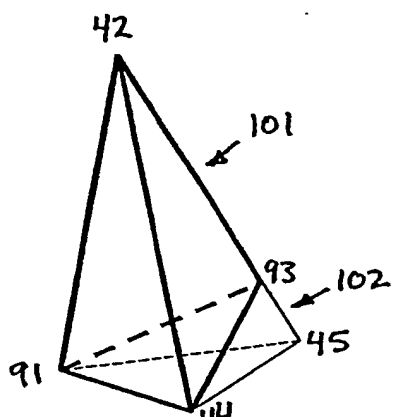
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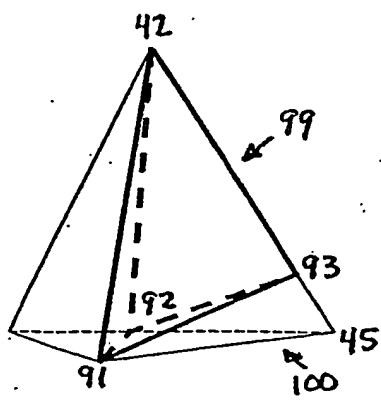
Fig. 11C
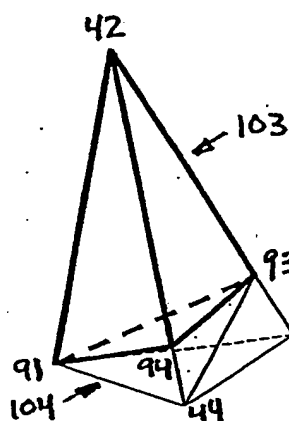
Fig. 12C
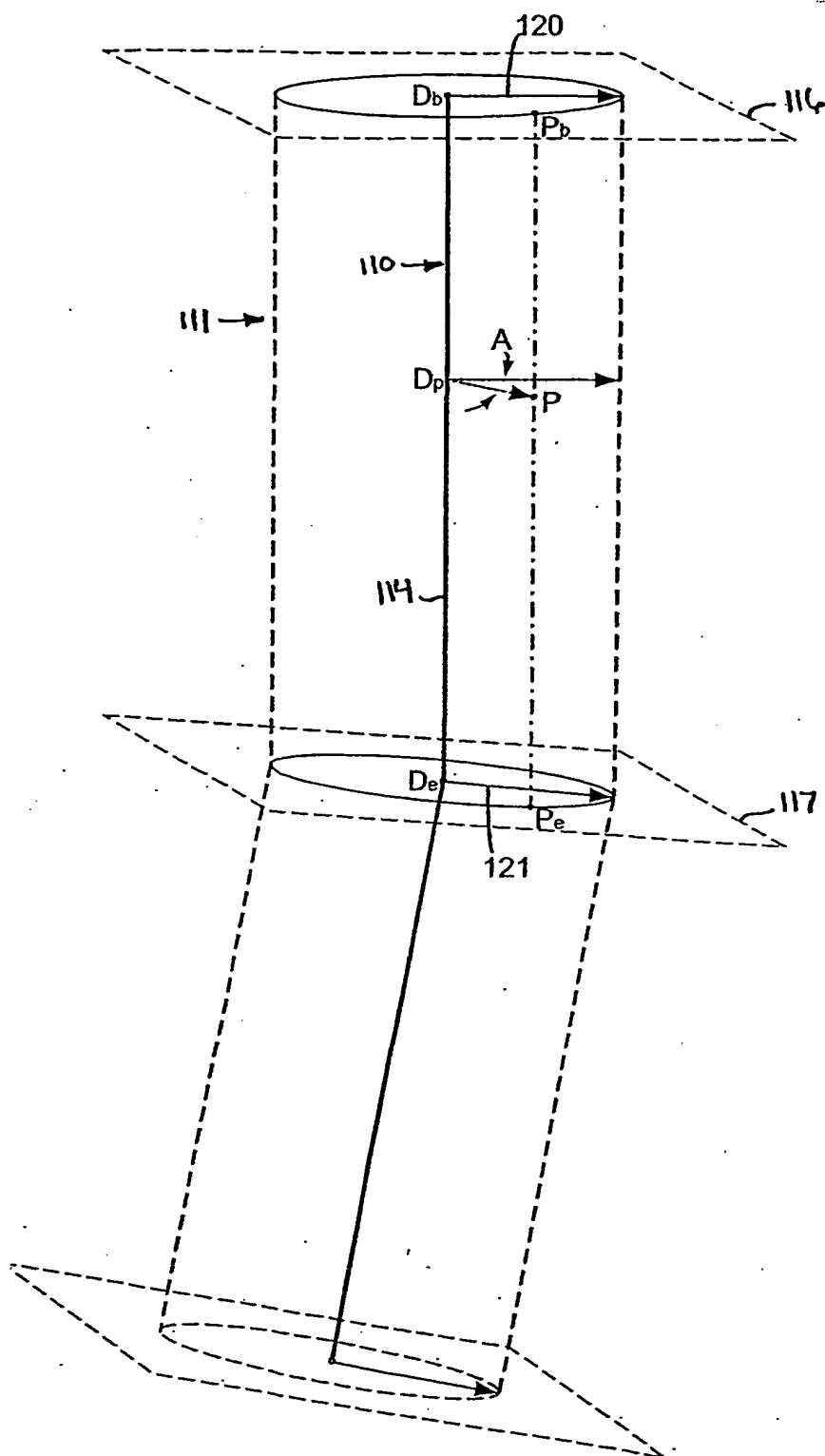
Fig. 13

INTERNATIONAL SEARCH REPORT

A. CLASSIFICATION OF SUBJECT MATTER
IPC 7 G01V11/00

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 7 G01V G06T

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

COMPENDEX, EPO-Internal, WPI Data

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category°	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	FORSYTH P A ET AL: "LOCAL MESH REFINEMENT AND MODELING OF FAULTS AND PINCHOUTS" SPE FORM EVAL JUN 1986, vol. 1, no. 3, June 1986 (1986-06), pages 275-285, XP002191830 page 279, column 1, line 1 - line 32 -/--	1,2,5,6, 10,11, 19,20, 23,24, 28,29

| X | Further documents are listed in the continuation of box C. | | X | Patent family members are listed in annex. |

° Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance

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"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
1 March 2002	22/03/2002

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 European Patent Office, P.B. 5818 Patentlaan 2
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 Fax: (+31-70) 340-3016 | Authorized officer

 Lorne, B |

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C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication,where appropriate, of the relevant passages	Relevant to claim No.
X	DING YU ET AL: "Development of dynamic local grid refinement in reservoir simulation" PROCEEDINGS OF THE 12TH SPE SYMPOSIUM ON RESERVOIR SIMULATION;NEW ORLEANS, LA, USA FEB 28-MAR 3 1993, 1993, pages 501-510, XP002191831 Proc SPE Symp Reservoir Simul;Proceedings of the SPE Symposium on Reservoir Simulation 1993 Publ by Soc of Petroleum Engineers of AIME, Richardson, TX, USA page 501, column 2, line 1 - line 18 page 502, column 1, line 14 -column 2, line 8 page 503, column 2, line 22 - line 29 ---	1,2,7, 10,19, 20,25,28
X	VERMA SANTOSH ET AL: "Control volume scheme for flexible grids in reservoir simulation" PROCEEDINGS OF THE 1997 SPE RESERVOIR SIMULATION SYMPOSIUM;DALLAS, TX, USA JUN 8-11 1997, 1997, pages 215-227, XP002191832 Richardson, TX, USA page 215, column 2, paragraph 2 -page 216, column 1, paragraph 1 page 220, column 1, paragraph 2 -page 222, column 1, paragraph 1 ---	1,16
X	US 6 078 869 A (GUNASEKERA DAYAL L) 20 June 2000 (2000-06-20) column 24, line 54 -column 26, line 3 column 33, line 8 - line 13; claim 1 ---	1,19
X	US 5 740 342 A (KOCBERBER SAIT) 14 April 1998 (1998-04-14) abstract; figure 4 ---	1,19
A	KOCBERBER SAIT: "Automatic, unstructured control volume generation system for geologically complex reservoirs" PROCEEDINGS OF THE 1997 SPE RESERVOIR SIMULATION SYMPOSIUM;DALLAS, TX, USA JUN 8-11 1997, 1997, pages 241-252, XP002191833 Richardson, TX, USA page 241, column 2, line 17 -page 242, column 1, line 20 ---	1,16,19

-/--

2

International Application No

PCT/US 01/20908

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

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